Chp 13 HW key

13.1

Exercise 13.1 *Problem 15.1.10 Boas (2006). A shopping mall has four entrances, one on the North, one on the South, and two on the East. If you enter at random, shop and then exit at random, what is the probability that you enter and exit on the same side of the mall?*

There are 4 entrances N, S, Ea, Eb. We assume equal probability of leaving or exiting by each (uniform distribution). Thus, the possible equal likelihood, mutually exclusive combinations are:

	N	S	Ea	Eb
N	*NN*	NS	NEa	NEb
S	SN	*SS*	SEa	SEb
Ea	EaN	EaS	*EaEa*	*EaEb*
Eb	EaN	EaS	*EaEa*	*EaEb*

where rows are entering and column are exiting. Italics denote the positive outcome (enter and exit on same side). Thus, the probability of entering and exiting on the same side is 6/16.

13.2

Exercise 13.2 *Problem 15.3.15 Boas (2006). Don't use the reduced sample space, just use Bayes' theorem. Use Bayes' formula (3.8) to repeat these simple problems previously done by using a reduced sample space. (a) In a family of two children, what is the probability that both are girls if at least one is a girl? (b) What is the probability of all heads in three tosses of a coin if you know that at least one is a head?*

a) The probability of child A being a girl is $P(A) = 0.5$. The probability of child B being a girl is $p(B) = 0.5$. We assume these are independent ($p(AB) = P(A) \cdot p(B)$), and there are 4 equally likely outcomes: 2 girls, 1 boy 1 girl, 1 girl 1 boy, 2 boys. $P(A + B)$ is the probability of at least one girl, $p(A+B) = p(A)+p(B)-p(AB) = 3/4$. $p(AB)$ is the probability of 2 girls, $p(AB) = 1/4$. Thus, Bayes gives the probability of 2 girls (AB=true) when at least one is a girl (A+B=true) as $p_{A+B}(AB) = p(AB)/p(A + B) = 1/4/(3/4) = 1/3$ is the probability of two girls given at least one is a girl. The state space estimate from the previous version was just to note only one out of three options: 2 girls, 1 boy 1 girl, 1 girl 1 boy, but the assignment was to use Bayesian manipulation.
b) The probability of heads is 0.5. We assume all tosses are independent, and there are 8 equally likely outcomes. $p(A)$ is probability of heads on first, $p(B)$ on second, $p(C)$ on third. $p(ABC)$ is the probability of all heads, $p(ABC) = p(A) \cdot p(B) \cdot p(C) = 1/8$. $p(A+B+C)$ is the probability of at least one head, which occurs for all equally likely outcomes except HHH, so $p(A+B+C) = 7/8$. Thus, $p_{A+B+C}(ABC) = p(ABC)/p(A + B + C) = 1/8/(7/8) = 1/7$.

13.4

Exercise 13.4 *Make up a small dataset of 10 or so data. a) Calculate the mean, variance, and standard error of the mean. b) Describe what statistics of the data are expected to have a distribution where the variance describes the spread and what statistics are expected to have a distribution where the standard error describes the spread of the statistic. c) Describe how jackknife estimation and bootstrap estimation can be used to produce a histogram categorizing the uncertainty in the mean. d) (optional) You may carry out the bootstrap and jackknife estimates computationally for extra credit.*

dataset: 7.0605 0.3183 2.7692 0.4617 0.9713 8.2346 6.9483 3.1710 9.5022 0.3445. a) mean= 3.9782, variance= 13.0080, standard error = $\sqrt{13.0080}/\sqrt{10}$ = 1.1405. b) The mean and variance of this data (generated from a random uniform distribution in matlab) describe the spread of the data points themselves, and would describe the histogram of more points drawn from the same distribution. The mean and standard error give the statistics of the *sample mean* of 10 points averaged together. This distribution is more normal (Gaussian) than the original distribution and also more narrow (smaller standard deviation by $\sqrt(10)$). c) In this case, there are 10 jackknife estimates of the mean, which are: 3.6357 4.3848 4.1125 4.3689 4.3123 3.5052 3.6481 4.0678 3.3644 4.3819. Each one is the average of 9 of the original data leaving one value out. A histogram can be made of these data, and it is clear that they cluster roughly around the mean of the original distribution with roughly the right standard error (mean of jackknife estimates is the same as sample mean, 3.9782, but in this case the variance of the jackknifes is much smaller than predicted by central limit theory 13.0080/10 0=1.30. Jackknife variance is 0.1606). To form a distribution using bootstrapping, we randomly choose sets of 10 data from the original data with replacement (i.e., repeated values are OK). Doing this, I found a mean of 3.9684, quite near the sample mean and a variances between 1.14 and 1.18. d) I did the manipulations using matlab functions jackknife and bootstrp: e.g., mean(jackknife(@mean,A)) var(jackknife(@mean,A)) mean(bootstrp(10000,@mean,A)) var(bootstrp(10000,@mean,A)).

13.6

Exercise 13.6 *Find the zeroth, first, and second moments (not normalized or centralized) of the continuous uniform distribution. Use these to derive the mean and variance of the continuous uniform distribution given in Table 13.2.*

The continuous uniform probability density function is constant over the interval of possible values (here taken to be $a \le x \le b$) as in Table 13.2. Thus,

$$\begin{aligned}
\rho(x; a, b) &= \frac{1}{b-a}, \\
\langle x^0 \rangle &= \int_a^b x^0 \frac{1}{b-a}\,\mathrm{d}x = \int_a^b \frac{1}{b-a}\,\mathrm{d}x = 1, \\
\langle x^1 \rangle &= \int_a^b x^1 \frac{1}{b-a}\,\mathrm{d}x = \int_a^b \frac{x}{b-a}\,\mathrm{d}x = \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2}, \\
\langle x^2 \rangle &= \int_a^b x^2 \frac{1}{b-a}\,\mathrm{d}x = \int_a^b \frac{x^2}{b-a}\,\mathrm{d}x = \frac{b^3-a^3}{3(b-a)},
\end{aligned}$$

Then the mean and variance are

$$\begin{aligned}
\langle x \rangle &= \langle x^1 \rangle = \frac{b+a}{2}, \\
\langle x^2 \rangle - \langle x \rangle^2 &= \frac{b^3-a^3}{3(b-a)} - \frac{(b+a)^2}{4} = \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4} = \frac{(a-b)^2}{12}.
\end{aligned}$$

13.7

Exercise 13.7 *Make a Venn diagram that describes an aspect of your life or work. Does it reflect independence or mutual exclusivity?*

I'm pretty happy with the ones in Fig. 13.3. Fig. 13.3b is a nice example of mutual exclusivity.